1. *Location choice* As an owner of a bubble tea business you are choosing between two locations for opening a new store. Your operating profit margin (before fixed costs, such as equipment, staff training etc.) is $.80 per cup of tea sold. Which location, 1 or 2, maximizes expected profits? Assume a 1-year horizon for the full depreciation of any fixed costs (e.g. equipment).

Location 1: Demand is uniformly distributed between 3500 and 5000 cups per month

Location 2: Demand is uniformly distributed between 2000 and 5000 cups per month

Location 1: You need to buy new equipment, which you estimate to cost $5000 with probability 40% and $7000 with probability 60%.

Location 2: There is a possibility (50%) that you can purchase equipment at a reduced price of $1000 from the coffee shop currently renting the space. Otherwise, you have to buy new equipment at the same cost as at location 1.

**Solution:**

**GAP words:**

**O:** maximize profits (Operating profit – fixed cost)

**D:** Which location to choose

**R:** demand, equipment costs, possibility of getting used equipment.

**GAP math:**

**D:** Location 1 or Location 2?

**R:** D1: Monthly demand at location 1, U[3500,5000]  
 D2: Monthly demand at location 2, U[2000,5000]  
 C: fixed cost of purchasing new equipment, {5000, 7000} with probabilities {0.4,0.6}  
 U: able to get used equipment? U=1 with probability 0.5, U=0 otherwise.  
**O:** MAX {Profit\_1, Profit\_2}, where  
 Profit\_1 = 0.8 \* D1 \* 12 – C  
 Profit\_2 = 0.8 \* D2 \* 12 – U\*1000 – (1-U)\*C

Note: we multiply monthly demand and profit margin by 12 to scale it to 1-year horizon.

**Answer:** Average profit atLocation 1 is around $34,600, while average profit at location 2 is Location 1 is around $30,000. Thus, location 1 is preferred to location 2. See Excel file for details.

*2) Profit Analysis:* A consumer electronics firm produces a line of battery rechargers for cell phones. The following distributions apply:

|  |  |
| --- | --- |
| Unit Price | Discrete uniform with possible prices of $23, $24, $25, and $26 |
| Unit Cost | Continuous uniform with a minimum of $12.00 and a maximum of $15.00 |
| Quantity Sold | 10,000 – 250\*Unit price, plus a random term given by a normal distribution with a mean of 0 and a standard deviation of 10 |
| Fixed Costs | Normal with a mean of $30,000 and a standard deviation of $5,000 |

1. What is the maximum loss?
2. What is the expected profit?
3. What is the probability of a loss?
4. Assume now that you are a monopolist and can unilaterally set prices (That is, unit prices are no longer uniform random variables but are determined by you). To maximize expected profit, what unit price would you set?

**Solution:**

**GAP words:**

**O:** Parts a-c:Estimate the loss/profit/prob of loss  
 Part d: maximize profits (Revenue – variable cost – fixed cost)

**D:** Parts a-c: We are not deciding anything (we are “price-takers”). Part d: Unit price

**R:** price, variable cost and fixed cost are all uncertain.

**GAP math:**

**D:** Parts a-c: **-,** Part d:p

**R:** p = price, discrete uniform {23,24,25,26}  
 c = unit cost, uniform [12,15]  
 q = quantity sold, 10000-250\*p + r, where r is Normal (0,10)   
 C = fixed cost, Normal(30,000, 5,000)  
**O:** Part d: Maximize (10000 – 250\*p + r)\*(p – c) – C

1. Approximately $10000, very volatile (See Excel file for details)
2. Approximately $12250 (See Excel file for details)
3. Approximately 2.4% (See Excel file for details)
4. Price p=27 results in the highest expected profit (See Excel file for details)

*Note that in part b), if we replace all random quantities by the averages, we will get an incorrect value for average profit: (10,000 – 250\*24.5)\*(24.5-13.50)-30000 = $12625. Thus, if we rely on averages instead of solving this problem using simulation modeling, we would be overestimating our profits by $12625-$12,250=$375.*

*3) Cashflow Analysis:* Vinton Auto Insurance is deciding how much money to keep in its checking accounts to cover insurance claims. In the past, the company held some of the premiums it received in interest-bearing checking accounts and put the rest into investments that are not quite as liquid, but tend to generate a higher investment return. The company wants to study cash flows to determine how much money it should keep in its checking accounts to pay claims. After reviewing historical data, the company has determined that the number of repair claims filed each week is a random variable that follows the probability distribution shown in the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Number of Claims** | 0 | 1 | 5 | 10 |
| Probability | 0.1 | 0.5 | 0.3 | 0.1 |

The company has also determined that the **average** repair bill per claim is normally distributed with a mean of $1,200 and standard deviation of $300. To be clear, the repair bills of each individual claim are **not** normally distributed with a mean of $1,200 and a standard deviation of $300. Rather, the **average** repair bill of a batch of claims for a given week is normally distributed with a mean of $1,200 and a standard deviation of $300. In addition to repair claims, the company also receives claims for cars that have been “totaled” and cannot be repaired. There is a 15% chance of receiving one claim of this type in any week, and there is no chance of receiving more than one in any week. The repair bills for “totaled” cars is given by the following: $7500 \* X, where X is a log-normal random variable with a mean parameter of .15 and a standard deviation parameter of 0.5.

1. Develop a simulation model and report the mean total cost of all claims incurred by the company in any week.
2. Suppose that the company decides to keep $15,000 cash on hand to pay claims. What is the probability that this amount will *not* be adequate to cover claims in any given week?
3. What level of cash would the company have to have to be 97% certain they could pay all the claims in any given week?

**GAP words:**

**O:** a) estimate total cost, b) estimate prob. That amount not adequate, c) estimate the level of cash required for 97% coverage.

**D:** Part a,b: no decisions.Part c: Level of cash

**R:** Number of claims, Average repair bill, Number of “totaled” claims, Repair bill for totaled

cars.

**GAP math:**

**D:** Part c) H = level of cash held by the firm

**R:** NC = Number of claims {0,1,5,10} with probabilities (0.1, 0.5, 0.3, 0.1)  
 C = Average claim, Normal (1200,300)  
 Totaled claim? T = 0 with probability 85%, T = 1 with prob. 15%.  
 Bill for totaled: TB = 750 \* Lognormal (0.15, 0.5)  
**O:** Part c) Prob (Total cost <= H) as close as possible to 0.97, or MIN |Prob(Total cost<=H) – 0.97|  
 where Total cost = NC \* C + T \* TB

1. See Excel file
2. Approximately 7% (See Excel file for details)
3. Approximately $19500 (See Excel file for details)

*4) Production Planning with Returns.* A computer manufacturer sells its laptop model through a web-based distributor, who buys at a unit cost of $200 and sells at a unit price of $500. The product life cycle is so short that the distributor is given only one opportunity to order stock before the technology becomes obsolete and a new model becomes available. At the beginning of the cycle, the distributor orders a stock level in the face of uncertain retail demand. Based on similar experiences in the past, the distributor believes that a reasonable demand model is a uniform distribution with a minimum of 1,000 and a maximum of 8,000 laptops. The items originally stocked are ultimately sold, returned, or scrapped. Customers place orders on the Web, and the distributor tries to satisfy their orders from stock. If there is a stockout, demands are lost.

The computer manufacturer offers the distributor a returns policy of the following form: It will pay $100 for each returned unit at the end of the product life cycle, but only up to a maximum of 20 percent of the original number of units ordered. Excess stock that cannot be returned to the manufacturer is picked up as scrap material by an electronics recycling center, with no cost or revenue involved. The decision facing the distributor is to choose an appropriate stock level.

1. Suppose there is no ceiling on the return of excess laptops. How many laptops should the distributor stock in order to maximize its expected profit?
2. With the returns ceiling in place, how many laptops should the distributor stock?
3. In part (b), what would be the maximum expected profit for the distributor?
4. What would be the corresponding expected profit for the *manufacturer* if the manufacturing cost is $125 per laptop, and the distributor uses the policy in part (b)?

**GAP words:**

**O:** Maximize distributor profit (Revenue – cost)   
**D:** Number of laptops to stock  
**R:** Demand for laptops

**GAP math:**

**D:** X = Number of laptops stocked by distributor

**R:** D = Demand, uniform [1000, 8000]  
**O:** part a: Max Profit = min(X,D) \* (500-200) – max(0,X-D)\*(200) + max(0,X-D)\*(100)

part b: Max Profit = min(X,D) \* (500-200) – max(0,X-D)\*(200) + max(0,min(X-D,X\*0.2))\*(100)

*Note:*

*The distributor’s objective function includes three parts: Revenue minus the Cost of overstocking, plus Returns from the manufacturer (if any).*

*The objective function looks more complicated than it is (and may be less intuitive than the Excel file with the simulation model). Essentially, min(X,D) means that we earn the profit margin on either the amount we stock, or the demand, depending on which one is lower. In other words, if we stock more than we can sell, we earn the profit margin on what we can sell, and if we stock less than we can sell, we only earn the profit margin on what we have stocked.*

*Similarly, max(0,X-D) means that if X>D, i.e. we have stocked more than demand, the multiplier is X-D. So we have to pay unit cost times X-D. Otherwise, i.e. if X<D, we have stocked less than demand, and have no cost of overstocking.*

1. **Around 6,200 orders** (See Excel spreadsheet for details)
2. **Around 5,500 orders** (See Excel spreadsheet for details)
3. **Around $980,000** (See Excel spreadsheet for details)
4. Step 1 Sell to Distributor

Sales = ordered

Step 2 Reimburse Distributor

Reimbursements = actual\_return

Step 3 Calculate Profit

($200 price - $125 Cost per computer) \* ordered - $100 \* actual\_return

Answer: **Around $350,000** (See Excel spreadsheet for details)

*5) Coffee Shop (HARD).*

Suppose you want to open a coffee shop. Each minute you have 0, 1, 2, or 3 customers with equal probabilities. It takes one espesso machine one minute to make one coffee. You charge $4 for one espresso drink. If the queue (line) is longer than 5 people, customers leave the shop. Operating costs of one espresso machine are fixed and equal $50000 per year, including labor costs of a barista. Your planning horizon is 1 year, which includes 300 days p. year and 8 hours p. day

1. Should you purchase one or two espresso machines?
2. Suppose you have overestimated demand. The true demand is 0, 1 or 2 customers with equal probabilities. Should you purchase one or two espresso machines?

**GAP words:**

**O:** Maximize profit  
**D:** 1 or 2 espresso machines  
**R:** Number of customers each minute

**GAP math:**

**D:** N = 1 if 1, N=2 if 2 espresso machines  
**R:** D = *n* = {0,1,2,3} # customers served, discrete uniform

customers who abandon each minute

**O:** Max Profit =

Answers to a) and b): See EXCEL solution.